

*Interdisciplinary Journal of
Research on Religion*

Volume 5

2009

Article 3

Religion and Faith:
A Decision Theory Perspective

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Abstract

We examine the implications of decision theory for religious choice and evangelism under the assumption that people choose their religion. The application of decision theory leads us to a broad definition of religion and a particular definition of faith, each related to the uncertainty associated with what happens to a person after death. We examine two extremes: total ambiguity and no ambiguity. For the extreme of total ambiguity, we show that there is “designer religion,” which is a religion that will capture all decision makers when any one of the standard decision criteria is applied. For the extreme of no ambiguity, we characterize the conditions under which a decision maker will find new religious information more valuable, and we characterize a “miracle” in a specific way.

The foresight of our own dissolution is so terrible to us . . . [it] makes us miserable while we are alive The dread of death [is] the great poison to happiness It afflicts and mortifies the individual.

Adam Smith (2000 [1759]: Part I, paragraph I.I.13)

I know . . . I must soon die, but what I know least is the very death I cannot escape.

Blaise Pascal (1958 [1670]: fragment 194)

What happens to you when you die? Adam Smith emphasizes the importance of the question, while Blaise Pascal emphasizes the uncertainty attending the answer. One answer is that your death is simply the end of you. However, religions throughout history have offered other answers. We examine the implications of decision theory for religious choice and evangelism under the assumption that people choose their religion.¹

We begin, in the next section of this article, by defining religion in a way that allows decision theory to be applied. Laurence Iannaccone (1998) identifies defining religion and characterizing its substance as a gap in the literature on the economics of religion, so our definition may be a contribution in itself. In their seminal paper, Azzi and Ehrenberg (1975) distinguish religion by its promise of afterlife rewards. Traditionally, religion has also been characterized by a belief in one or more gods. The definition that we propose is slightly more general. We conceive of a religion as an understanding of what happens to a person after death, accompanied by an associated set of prescriptions for how to live life. This understanding seems to be the essence of religion from a decision theory perspective because it seems to be the primary factor that shapes a decision maker's perception of the payoffs that are associated with various religious alternatives.

A particular conception of religious faith naturally follows from the definition of religion. Faith is the act of choosing one religion, or one idea about the after-

¹ Many individuals do not accept the idea that people choose their religion. Within Christianity, for example, different sects have arisen because of different answers to the question "Do you choose God, or does God choose you?" In the 1552 "Consent" concerning the "Eternal Predestination of God . . .," Protestant reformer John Calvin writes, "it ought to have entered the minds . . . from whence faith comes; . . . it flows from Divine election as its eternal source" (Calvin 2007 [1552]). The Calvinist perspective, which is also associated with the Catholic theologian Augustine, is that God chooses you. The prototypical Christian alternative to Calvinism is Arminianism, which is associated with the Dutch theologian Jacob Hermann, best known by his Latin name Jacob Arminius. The Arminian doctrine is that God's grace is offered to all, but one can refuse it. In other words, people must choose God. Other religions around the world and sects within them vary in their beliefs about one's ability to choose a faith or religion. We intend no disrespect to any faith as we construct a model under the assumption that one chooses one's religion. This assumption might be wrong.

life, over the other alternatives, with the recognition that the choice might be incorrect. Religion may involve faith that there is a supernatural being or faith that there is not. It may involve faith that there is an afterlife or faith that there is not. From this perspective, atheism is religion, requiring faith, just like any well-known theism.

Science cannot effectively help us to understand what happens to us when we die. There is no generally accepted database on after-death experiences. *Bartlett's Familiar Quotations* was first published in 1892, was most recently revised in 2002, and is organized chronologically. The first quotation found in the 2002 edition (Kaplan 2002: 3) is "The Song of the Harper," which is an Egyptian quotation translated from the tomb of King Inyotef and dating to 2650–2600 B.C.:

There is no one who can return from there,
To describe their nature, to describe their dissolution,
That he may still our desires,
Until we reach the place where they have gone.
Remember: It is not given to a man to take his goods with him.
No one goes away and then comes back.

This ancient quotation indicates that people have always struggled with the fact that if there is life after death, we do not get much of a glimpse of it while we are alive. We can readily develop theories that explain what happens after death. However, we cannot readily reject even the most incredible theory by testing it against the facts.

If one must choose a religion without the benefit of science, how should one choose? This question was asked and answered in the 17th century by Blaise Pascal. Known for his genius when he was a child and for his many contributions to mathematics and science, Pascal focused much of his attention late in his short life on questions of religion. He argued that "prudence" can provide insight that scientific "reason" cannot. Notably, in formulating and solving a choice problem that has become known as *Pascal's wager*, he brought about the advent of decision theory (Jordan, 1994a).

In its simplest form, Pascal's wager can be thought of as a choice between theism and atheism. In Pascal's words (Pascal 1958 [1670]: fragment 233):

God is or He is not. But to which side will we incline? . . . What will you wager? . . . You must wager. It is not optional . . . Let us weigh the gain and the loss in wagering that God is. . . . If you win, you win everything, if you lose you lose nothing. Do not hesitate then; wager that he does exist.

In this version of his wager, Pascal assumes that theism weakly dominates atheism. Consequently, prudence demands that theism be chosen over atheism.²

Of course, theism might not weakly dominate atheism in the mind of a decision maker, and Pascal recognized this. So he also presented his wager as a dilemma. We review this dilemma formulation, assuming total ambiguity. When there is any ambiguity, the decision maker's beliefs cannot be represented by a unique objective or subjective probability distribution over the states of nature.³ Under total ambiguity, the decision maker perceives every probability distribution over the set of states of nature as being possible. A number of decision criteria have been proposed for describing how a decision maker can resolve dilemmas and rationally make a choice under total ambiguity. These include the max-max criterion, the max-min criterion, the Hurwitz criterion, the min-max regret criterion, and the principle of insufficient reason. As our primary result, we show that for total ambiguity, there is "designer religion," one that will capture any decision maker when any one of these standard criteria is applied. The implication for evangelism, as we explain further below, is that the evangelist, seeking to attract those who view the religious choice as highly ambiguous, should stress the potential rewards for adopting the religion and the potential penalties for non-adoption.

We then examine what we can learn by assuming no ambiguity, in which case the decision maker has subjective expected utility theory (SEU) preferences. Montgomery (1996) rightly questions the usefulness of SEU, the opposite extreme of the total ambiguity case, arguing that objective religious information is so lacking that it is not likely that a decision maker can form a unique probability distribution over states or rationally update these beliefs. However, we find that studying the no ambiguity case does offer some insights. For example, we are able to identify when a decision maker should be more interested in obtaining religious information, say, from a sermon, and when the decision maker should be less interested. We are also able to use standard Bayesian updating to characterize a religious miracle, distinguishing it from other events.

We conclude by highlighting the implications of our results for evangelism, and we discuss how the simple framework that we present here can be extended.

²Atheists reading this might be offended by the suggestion that an atheistic belief is imprudent. Morris (1994) suggests that offending atheists, or at least making them think twice about their unbelief, might have been Pascal's intention, possibly in response to being questioned by atheists about the prudence of his faith. Speaking of atheists, Pascal wrote, "Do they think they have given us great pleasure by telling us that they hold our soul to be no more than wind or smoke, and saying it moreover in tones of pride and satisfaction?" (Morris 1994: 51–52).

³ See Epstein (1999) for a more in-depth discussion.

DEFINING RELIGION AND FAITH: A DECISION THEORY PERSPECTIVE

To apply decision theory, we must organize our thinking about religion in a particular way and construct a decision matrix. Our approach is to assume that the alternative choices in the matrix are alternative religions that the decision maker (DM) perceives, while the alternative states are different perceived “truth” possibilities.

A *religion* is an understanding of what happens to a person after death, accompanied by a set of prescriptions for how to live life. Applying this definition, DM perceives n religious alternatives and perceives that these are mutually exclusive. The perceived mutual exclusivity implies that DM also perceives that one and only one religion $j \in (1, 2, \dots, n)$ is true. *Faith* is the act of adopting a religion, choosing it over the other alternatives. DM perceives that the payoff u_{ii} will be received when religion i is adopted and religion j is true. Implicitly, this means that DM perceives that adopting a religion has consequences, both when it is true and when it is not, and that DM can map these consequences into perceived payoffs.

This decision theoretic perspective of religion has some attractive features. First, it is a very general notion of religion, allowing the inclusion of traditional religions and nontraditional religions. Even atheism is a religion in this view, because it too is a perspective about the afterlife, shrouded in uncertainty like any other religion, which can be mapped in DM’s mind D to potential payoffs.

Second, faith in the model is consistent with commonly held notions of faith. In Hebrews 11:1, the Bible describes faith as the “substance of things hoped for, the evidence of things not seen.” When DM adopts religion i , this choice is the outward sign or substance of the hope that religion i is true, the hope that utility u_{ii} will be received, or the hope that some lower utility level u_{ij} , where $j \neq i$, will not be received. McCloskey (2006) describes faith as spiritual courage but notes that courage also requires faith. In our model, DM displays courage by adopting a religion because DM does not know the truth with certainty and adopting a false religion would generally be expected to have negative consequences. Yet to display courage, DM must “keep the faith” and endure this uncertainty.

It might seem odd to think that all decision makers display spiritual courage and faith, no matter what religious choice they make, but this was a fundamental point made by Pascal, who said, “You must wager [as to whether or not God exists]. It is not optional” (Pascal 1958 [1670]: fragment 233). So, for example, choosing atheism requires spiritual courage, just as choosing a theism requires courage. Of course, agnosticism is, by definition, choosing not to choose a religion, so one might think that agnostics lack courage. However, in Pascal’s mind, choosing not to choose is also a choice that has consequences, an indication that it is possible to rationalize agnosticism by including it as a row in DM’s

decision matrix.⁴ Whether the choice from the decision matrix is agnosticism, atheism, or a particular theism, DM displays conviction under uncertainty about what happens after death, something that takes a degree of spiritual courage and faith.⁵

RELIGIOUS CHOICE UNDER TOTAL AMBIGUITY

If one religion strongly dominates all alternatives in DM's mind, then what happens after death, while uncertain, is inconsequential. However, for many decision makers, no religion is even weakly dominant. McClennan (1994: 188) remarks, "Many will reject the claim that, if God does not exist, you have lost nothing by betting on God. Betting on God risks something of value, if only the experience of the more sensual pleasures to which, it is typically alleged, a nonbeliever will be drawn." Pascal recognized this through his straw man skeptic, who responded to Pascal's initial wager by saying, "Yes, I must wager, but perhaps I wager too much" (Pascal 1958 [1670]: fragment 233). If weak dominance does not hold, the wager becomes a dilemma.

In response to skeptics, Pascal presented his wager as a dilemma. In his words, the modified payoff structure involved an "infinity of infinite life and happiness to be won" when you bet on God, while "what you are staking is finite" (Pascal 1958 [1670]; fragment 233). Sorensen (1994: 140) colorfully translates this payoff structure as being one in which "God assigns believers infinite bliss and unbelievers infinite blitz." Sorensen (1994), Tabarrok (2000), and Osterdal (2004) each discuss interesting issues that arise when infinite payoffs are perceived. We avoid these issues here by assuming that any perceived payoff is finite. The St. Petersburg Paradox informs us that people may ascribe finite values to infinite payoffs,⁶ so we have an empirical basis for assuming that people

⁴ Agnosticism can also be rationalized as a choice that is a function of the alternative religions in the decision matrix, and we will present such a model of agnosticism in a companion article.

⁵ Of course, the concept of spiritual courage does not mean much if everyone displays it—theists, atheists, and agnostics. While we do not pursue this line of thinking in detail here, the decision theory concept of regret could be used to measure spiritual courage. A decision maker who minimizes the maximum regret could be labeled as displaying the least courage, while criteria that allow more potential regret could be labeled more courageous.

⁶ Presented by Nicholas Bernoulli in 1713, the St. Petersburg Paradox is based on the idea that people are not willing to pay an infinite amount to play a game that produces infinite rewards. Bernoulli's game was to flip a coin and get 1 (dollar) if the outcome is heads; then flip a coin twice and get 2 (dollars) if each outcome is heads; then flip a coin n times and get 2^{n-1} (dollars) if each outcome is heads. Daniel Bernoulli offered the first solution to this paradox by inventing the idea of diminishing marginal utility. The Allais-Wierich solution involves discounting more heavily rewards that are received with a lower probability (Sorensen 1994). In either case, an infinite number of arbitrarily large but finite rewards can be perceived to have finite value if the marginal value of the successive rewards is discounted.

ascribe finite values to perceived religious payoffs. However, more significantly, we do not need to assume infinite payoffs to capture the spirit of Pascal's argument and derive other meaningful results.

As a starting point, consider a dilemma version of Pascal's wager as the choice between two religions. Assume that religion 1 (theism) is characterized by the understanding that there is life after death because God exists, and assume that religion 2 (atheism) is characterized by the understanding that there is no life after death because God does not exist. Assume that the decision maker perceives each of these afterlife perspectives as being accompanied by particular prescriptions for how to live life such that, when life and afterlife are considered, the decision maker perceives the payoffs shown in Table 1. The assumption that $u_{11} > u_{21}$ and $u_{22} > u_{12}$ makes the decision a dilemma.

Table 1: Dilemma Version of Pascal's Wager

| | Religion 1 True (God Exists) | Religion 2 True (God Does Not Exist) |
|-----------------------------|---|---|
| Religion 1 (Theism) | Afterlife bliss = u_{11} | Earthly sacrifice = u_{12} |
| Religion 2 (Atheism) | Afterlife blitz = u_{21} | Earthly pleasure = u_{22} |

It has been suggested (see, for example, Black 1997; Hansson 2005) that when there is total ambiguity, decision makers may resolve this dilemma by applying one of the following criteria:

- Max-max criterion
- Max-min criterion
- Hurwicz criterion
- Min-max regret criterion
- Principle of insufficient reason

It is useful to ask, "Under what additional assumptions will DM choose theism over atheism?" The max-max criterion is an optimistic approach, sometimes labeled *wishful thinking*. With $u_{11} > u_{21}$ and $u_{22} > u_{12}$, the optimist, seeking the maximum of the maxima, will choose theism as long as $u_{11} > u_{22}$. Thus when DM is an optimist, theism attracts DM if and only if $u_{11} > u_{22}$.

The max-min criterion is a pessimistic approach, sometimes labeled *prudent thinking*. With $u_{11} > u_{21}$ and $u_{22} > u_{12}$, the pessimist, seeking to maximize the minimum payoff, will choose theism as long as $u_{21} < u_{12}$. Thus when DM is a pessimist, theism attracts DM if and only if $u_{21} < u_{12}$.

The Hurwicz criterion (Arrow and Hurwicz 1972; Hurwicz 1952) parameterizes the degree of optimism/pessimism. With optimism level α , the Hurwicz

decision maker expects the payoff for theism to be $\alpha u_{11} + [1 - \alpha]u_{12}$, while the expected payoff for atheism is $\alpha u_{22} + [1 - \alpha]u_{21}$. The expected payoff for theism exceeds that of atheism if and only if $\alpha u_{11} + [1 - \alpha]u_{12} > \alpha u_{22} + [1 - \alpha]u_{21}$. This reduces to $u_{11} > u_{22}$ for $\alpha = 1$ and to $u_{21} < u_{12}$ for $\alpha = 0$, showing that the Hurwicz criterion includes the max-max and max-min criteria as special cases. If $u_{11} > u_{22}$ and $u_{21} < u_{12}$, then the best choice is theism, no matter what level of optimism $0 \leq \alpha \leq 1$ is held by a Hurwicz decision maker. That is, the conditions $u_{11} > u_{22}$ and $u_{21} < u_{12}$, which are needed to attract both the extreme optimist and extreme pessimist to theism, respectively, are together sufficient to attract all Hurwicz decision makers, regardless of the decision maker's degree of optimism.

The min-max regret rule captures the behavior of people who seek to avoid regret. If the decision maker mistakenly chooses theism when God does not exist, the payoff received is u_{12} , while the payoff that could have been received is u_{22} . Therefore the potential regret associated with choosing theism is $u_{22} - u_{12}$. Analogously, the potential regret associated with choosing atheism is $u_{11} - u_{21}$. The decision maker who seeks to avoid regret would choose theism if and only if $u_{11} - u_{21} > u_{22} - u_{12}$. Notice that the assumptions $u_{11} > u_{22}$ and $u_{21} < u_{12}$ are sufficient for the regret avoider to choose theism, though not necessary. The regret avoider would choose theism even when $u_{21} > u_{12}$, whereas pessimists would not, as long as $u_{11} > u_{22} + u_{21} - u_{12}$. Also, the regret avoider would choose theism even when $u_{11} < u_{22}$, whereas optimists would not, as long as $u_{21} < u_{11} - u_{22} + u_{12}$.

The principle of insufficient reason resolves the uncertainty by assuming that each state of the world is equally likely. Pascal gave some indication that this principle might be applied when he said, "A game is being played . . . where heads or tails will turn up" (Pascal 1958 [1670]: fragment 233). If we apply equal probabilities to the two states, the expected value of wagering that God exists is $\frac{1}{2}u_{11} + \frac{1}{2}u_{12}$, while the expected value of not wagering is $\frac{1}{2}u_{21} + \frac{1}{2}u_{22}$. Therefore it is best to wager on theism if and only if $u_{11} - u_{21} > u_{22} - u_{12}$. Thus in this context with only two religions, we find that the decision maker who applies the principle of insufficient reason behaves just like the decision maker who seeks to avoid regret.

These results are summarized by the following total ambiguity theorem, the proof of which is evident from the preceding analysis.

Total Ambiguity Theorem

If DM's beliefs are totally ambiguous, DM's preferences are given by the max-max criterion, the max-min criterion, the Hurwicz criterion, the min-max regret criterion, or the principle of sufficient reason; and if the payoff structure presents

a dilemma with $-\infty < u_{21} < u_{12} < u_{22} < u_{11} < +\infty$, then it is rational for the decision maker to choose religion 1 over religion 2.

One significant objection to the Pascal's wager as presented in Table 1 is the *many-gods objection*, which is the critique that many decision makers will perceive more than two religious alternatives.⁷ We can address this objection by adding additional religious alternatives to the decision matrix. This allows us to generalize the total ambiguity theorem, the proof of which is also straightforward.

General Total Ambiguity Theorem

Consider a DM who chooses among alternative religions (1, 2, 3, ..., n). If

- (a) Religion i is reward dominant ($u_{ii} > u_{jj}$ for all $j \neq i$) and
- (b) Religion i is penalty dominant ($\max_{k \neq i} (u_{ki}) < \min_{k \neq j} (u_{kj})$) for all $j \neq i$,

then a decision maker who has totally ambiguous beliefs and applies the max-max criterion, the max-min criterion, the Hurwicz criterion, or the min-max regret criterion will find it rational to choose religion i . The decision maker who applies the principle of insufficient reason will choose religion i if, in addition to conditions (a) and (b), the following condition holds:

- (c) Religion i is not differentially penalized ($\sum_{k \neq i, k \neq j} u_{ik} \geq \sum_{k \neq i, k \neq j} u_{jk}$ for all $j \neq i$).

Except for the application of the principle of insufficient reason, the general ambiguity theorem confirms the intuition that is obtained from the environment in which there are only two religious alternatives. To attract all optimists, a religion must be viewed as offering the highest payoff for adoption when the religion is true. Condition (a) is a requirement that this holds for religion i . To attract all pessimists, a religion must be viewed as imposing the highest penalty for nonadoption when the religion is true. Condition (b) is a requirement that this holds for religion i . Together, conditions (a) and (b) imply that religion i will also minimize the maximum regret that can be experienced.

⁷ See Jordan (1994b) for a discussion of the many-gods objection. Hacking (1994) and Morris (1994) suggest that Pascal did not intend for us to partition the set of choices into theism and atheism. Rather, they believe that it is more accurate to say Pascal's intent was to make the partition "pursuing God" and "not bothering about such things" (Hacking 1994: p. 25). Morris (1994: 6) contends that Pascal's intent was to use his wager "to put the unbeliever on the path to belief" in the Christian faith that Pascal had adopted. However, the many-gods objection remains as long as one allows the possibility that not all decision makers find the same religion in the search.

To attract all those applying the principle of insufficient reason, condition (c) must be added. In the state in which religion $k \neq i, j$ is true, the payoff for religion j may be higher than that for religion i . This could be, for example, because religion i is more costly to adopt in life than religion j is. In a sense, we can say that in the mind of DM, religion k penalizes religion i for being false more than it punishes religion j . If this differential penalty is large enough when accumulated over all of the religions k that might be true, then religion j could be rationally chosen over religion i even when $u_{ii} > u_{jj}$ and $u_{ji} < u_{ij}$. Condition (c) allows for differential penalties but assumes that religion i is not cumulatively penalized more than any another religion is. This condition, along with conditions (a) and (b), ensures that a decision maker who applies the principle of insufficient reason would choose religion i .

Because it would capture all decision makers under total ambiguity, we might label a religion that satisfies conditions (a), (b), and (c) a *designer religion* for total ambiguity. The analysis above informs us that the many-gods objection to Pascal's wager can have impact in three ways. First, the additional perceived religion may offer a newly perceived max-max outcome, eliminating an existing designer religion by attracting the optimists. Second, the additional religion may offer a newly perceived max-min outcome, eliminating an existing designer religion by attracting the pessimists. Third, adding a new religion offers more potential for differential punishment, which enhances the possibility that condition (c) will not hold and an existing designer religion would be displaced if the differential punishment were large enough.

RELIGIOUS CHOICE UNDER NO AMBIGUITY

Suppose that DM can form a probability distribution over the n perceived religion states. Let p_s denote the *belief* DM has in religion s , defined as the subjective probability that DM associates with religion s being true, where $p_s \geq 0$ and

$$\sum_{s=1}^{s=n} p_s = 1$$

When there exist at least two religions that have a strictly positive probability, religious belief is not all or nothing but rather is partial, reflecting DM's uncertainty about what happens after death. Alternatively, DM might be certain that one of the religions is true. In this case, all of the probability weight is allocated to one religious alternative.

The expected utility of religion i is

$$E_i = \sum_{s=1}^{s=n} p_s u_{is}$$

and the expected utility of religion i exceeds that of religion j if and only if

$$p_i[u_{ii} - u_{ji}] > p_j[u_{jj} - u_{ij}] + \sum_{k \neq i, k \neq j} p_k[u_{jk} - u_{ik}] \quad (1)$$

Condition (1) indicates that there are three factors that affect the attractiveness of religion i relative to religion j : the probability of truth, p_i compared to p_j ; the net payoff of the religion, $u_{ii} - u_{ji}$ compared to $u_{jj} - u_{ij}$; and the expected differential penalty

$$\sum_{k \neq i, k \neq j} p_k[u_{jk} - u_{ik}]$$

Thus one reason DM might not adopt religion i is that DM's belief in religion i is too low relative to some other religion j . However, having very little confidence that religion i is true is not sufficient to rule out religion i , for this shallow level of belief can be more than offset by a high net payoff relative to all other religions j . Thus abstracting from differential penalties, we can say that when there is no ambiguity, a particular religion is chosen either because DM perceives a high probability that the religion is true, relative to the alternatives, or because DM perceives a relatively high net payoff in comparison to all religious alternatives.

Differential penalties can alter this thinking. The quantity

$$\sum_{k \neq i, k \neq j} p_k[u_{jk} - u_{ik}]$$

is the expected differential penalty imposed by religions $k \neq i, j$ for adopting religion i when it is false rather than adopting j when it is false. If this penalty is positive, then the decision maker perceives that religions $k \neq i, j$ impose greater punishment on average for adopting religion i when it is false than for adopting religion j when it is false. This encourages the decision maker to adopt religion j rather than religion i . Thus while strong belief and a high perceived net reward encourage the adoption of a religion, possessing both is not sufficient when there are more than two religious alternatives. Here, we again see that the potential for differential punishment lends credence to the many-gods objection.

The expected value of perfect information informs us of the circumstances under which a decision maker would find religious information valuable and the circumstances under which he or she would not. The gross expected payoff for perfect information is given by

$$\sum_{s=1}^{s=n} p_s u_{ss}$$

whereas the expected payoff for religion i is

$$\sum_{s=1}^{s=n} p_s u_{is}$$

When religion i yields more expected utility than any other religion does, the net expected value of perfect information is

$$EVPI(i) = \sum_{s \neq i} p_s [u_{ss} - u_{is}] \quad (2)$$

Notice that the probability p_i is not in the $EVPI(i)$ sum. Thus condition (2) indicates that reliable religious information would be perceived as being more valuable as p_i decreases, or as the confidence DM has in the most attractive religion diminishes. In contrast, DM would not expect the value of new religious information to be high when the belief in religion i is very strong, that is, when p_i is high. There is an exception to this last statement, however. Even when the belief in religion i is strong, so that the probabilities p_s , where $s \neq i$, are small, DM would expect reliable new religious information to be valuable when the perceived net payoff $u_{ss} - u_{is}$ is very large for one or more of the religions, where $s \neq i$, that are not perceived to be best. So, for example, because Christianity and Islam both recognize the concepts of heaven and hell, the model indicates that Christians and Muslims who recognize the other religion as possibly true will be particularly interested in new religious information.

To consider how new information might alter beliefs, let I be an event (e.g., new information in a sermon or a personal experience) that leads the decision maker to reevaluate the probability distribution (p_1, p_2, \dots, p_n) . Let A_i denote the event that religion i is true, so $p(A_i)$ is the value of p_i prior to the event I . In other words, $(p(A_1), p(A_2), \dots, p(A_n))$ represents DM's prior beliefs. The assumptions that we have made imply that the religious truth space in the mind of the decision maker is partitioned by the events A_i , where $i = 1, 2, \dots, n$.

The posterior probability $p(A_i | I)$ is the decision maker's estimate for p_i after the event I has occurred. From the definition of conditional probability, we have that $p(A_i | I) = p(A_i \cap I) / p(I)$ and $p(I | A_i) = p(A_i \cap I) / p(A_i)$, where $p(I | A_i)$ is the likelihood function. One can rewrite Bayes' law as $p(A_i | I) = p(I | A_i) / p(A_i) / p(I)$. If we use this last condition, it follows that

$$p(A_i | I) - p(A_i) = \frac{p(A_i)[p(I | A_i) - p(I)]}{p(I)} \quad (3)$$

As long as $p(A_i) \neq 0$ and $p(I) \neq 0$, condition (3) immediately leads to the following observations:

- If $p(I/A_i) = p(I)$, so that the probability of event I is unaltered by conditioning on the truth of religion i , then the event I will not lead DM to update the religious belief p_i .
- When the truth of religion i is not independent of the event I , the decision maker will update the belief p_i .
- If $p(I/A_i) > p(I)$, meaning that DM perceives that event I would be more likely if religion i were true, then the posterior belief $p(A_i | I)$ in the truth of religion i will be greater than the prior belief $p(A_i)$.
- Alternatively, if $p(I/A_i) < p(I)$, meaning that DM perceives that event I would be less likely if religion i were true, then the posterior belief $p(A_i | I)$ in the truth of religion i will be less than the prior belief $p(A_i)$.
- The magnitude of the change in belief $p(A_i | I) - p(A_i)$ is relatively large when the event I is unlikely (i.e., when $p(I)$ is small).

We have that

$$p(I | A_i) = [1 + \varepsilon_i]p(I) \quad (4)$$

where

$$\varepsilon_i = \frac{p(I \cap A_i)}{p(I)p(A_i)} - 1$$

Here, $100\varepsilon_i$ is the percentage change in the decision maker's probability of experiencing event I as a result of knowing that religion i is true. ε_i parameterizes the decision maker's perceived degree of dependence between the truth of religion i and event I . ε_i is unrestricted in sign, so the presumed truth of religion i can either increase or decrease the probability of event I .

When $p(I \cap A_i) = 0$, that is, observing event I is inconsistent with the truth of religion i , ε_i assumes its smallest feasible value of -1 , while $p(I/A_i) = 0$. The largest possible value of ε_i is bounded above by

$$\min \left\{ \frac{1}{p(A_i)}, \frac{1}{p(I)} \right\} - 1$$

Thus

$$-1 \leq \varepsilon_i \leq \min\left\{\frac{1}{p(A_i)}, \frac{1}{p(I)}\right\} - 1 \quad (5)$$

Using the formula for conditional probability $p(A_i|I) = [p(I|A_i)p(A_i)]/p(I)$ and (4), we obtain

$$p(A_i|I) = [1 + \varepsilon_i]p(A_i) \quad (6)$$

Using conditions (5) and (6), we can characterize how occurrence of different events alters the strength of belief in religion i . When $\varepsilon_i = 0$ or, equivalently, event I is perceived to be independent of the truth of religion i , $p(I \cap A_i) = p(I)p(A_i)$, there is no adjustment in the strength of belief in religion i . When the decision maker cannot perceive anything but events that are independent of the truth of religion i , there is no information or any events that will alter his or her beliefs.

When

$$0 < \varepsilon_i \leq \min\left\{\frac{1}{p(A_i)}, \frac{1}{p(I)}\right\} - 1$$

the assumed truth of religion i makes event I more likely in the mind of the decision maker, so the occurrence of event I increases the strength of belief in religion i . We can label such an event a *miracle*. The magnitude of ε_i being a measure of the degree to which the event is considered miraculous. At the extreme $\varepsilon_i = [1/p(A_i)] - 1 \leq [1/p(I)] - 1$, the occurrence of the miracle (event I) increases the posterior belief $p(A_i|I)$ to 1, thus eliminating all other religions from the decision matrix.

Continuing to consider the special case $\varepsilon_i = [1/p(A_i)] - 1 \leq [1/p(I)] - 1$, we notice that the level that ε_i must assume to increase the posterior belief to 1 depends on the strength of the prior beliefs $p(A_i)$ and $p(I)$. When the decision maker already has a strong belief that religion i is true, a small positive value for ε_i (i.e., a “small miracle”) will be enough to move the decision maker’s belief to certainty. However, when the prior belief is weak, the decision maker is very skeptical of religion i , and a large positive value for ε_i is required (i.e., a “large miracle”).

When $-1 \leq \varepsilon_i < 0$, the assumed truth of religion i makes event I less likely, so the occurrence of event I decreases the strength of belief in religion i . In this case, event I is the antithesis of a miracle, which we might label a *nullification event*. At the extreme, when $\varepsilon_i = -1$ or, equivalently, $p(I \cap A_i) = 0$, the decision maker perceives that event I has zero probability of occurring when religion i is true.

Thus the occurrence of event I reduces the posterior belief $p(A_i | I)$ to 0. In this case, we might expect religion i to be removed from DM's decision matrix.

DISCUSSION

Iannaccone (1998: 1491) notes that "the problem of religious uncertainty has received little attention and scarcely any formal analysis." This is surprising in that religious uncertainty, at least as we have defined it here, is probably the quintessential example of uncertainty. Here, we have applied basic decision theory to construct a simple model of religious choice under uncertainty. We have examined two extreme cases: choice under total ambiguity and choice under no ambiguity. In doing so, we have generalized the theory of religious choice that Pascal first embodied in his wager (Pascal, 1958 [1670]: fragment 233).

Pascal's innovative idea was that while the extreme uncertainty of what happens after death precludes us from using scientific reason to derive what our religious choice should be, we can nonetheless derive a choice from logic. Even more controversial was his inference that we should all (perhaps) end up making the same religious choice. The model of religious choice that we have presented is general enough that it can support any particular observed religious choice as a rational choice while allowing us to understand why it is possible that all could arrive at the same optimal religious alternative.

Ambiguity, which one might expect to be significant in the case of religious choice, focuses a decision maker's attention on the perceived payoffs of the various religious alternatives. Under total ambiguity, optimists are attracted to religions that have high perceived rewards, while pessimists are attracted to religions that maximize the minimum perceived outcomes. If ambiguity is difficult to remove, the implication for evangelism is that stressing the rewards associated with adoption of the religion will be effective for attracting optimists, while stressing penalties for nonadoption will be effective for attracting pessimists. Adding more competing religions does not change this prescription for evangelistic success, though it does open the door for *differential punishment*, in which one religion's evangelistic success can be tempered when many other alternative religions single out that religion as being especially deserving of punishment. In our model, with total ambiguity, all will adopt one religion if that religion is perceived by all to offer the highest reward for correct adoption, perceived by all to yield the lowest potential penalty for incorrect nonadoption, and perceived by all not to be differentially punished by other religions to a strong degree.

If there are people who can manage religious uncertainty by assigning probabilities of truth across the various religious alternatives recognized, then our model under no ambiguity indicates that evangelism can be effective by

influencing these beliefs, in addition to influencing perceived outcomes. Such decision makers will have more room to be converted to a new religion by receiving new information when the net payoff of the new religion is larger. In stressing the net payoff, the evangelist would stress reward and punishment differentials, as in the case of total ambiguity. To alter beliefs, Bayesian updating indicates that the evangelist should highlight miracles, which are defined to be events that are not independent of the truth of the religion being promoted but rather are more likely to occur if the religion is true.

In this model, atheism in its various forms satisfies the definition of religion, for it is an understanding about what happens when one dies and has a set of associated prescriptions for how to live life. As with any other religion, it takes faith to adopt atheism in the face of the uncertainty as to what happens at death. Our model offers an explanation for atheism's relative unpopularity: It typically proposes neither large rewards for correct adoption nor extraordinary penalties for incorrect nonadoption. Therefore atheism will not tend to be chosen by decision makers who have sufficiently ambiguous beliefs. Moreover, to choose it under no ambiguity, one must be rather confident that one is correct.

In terms of critique, this simple model of religious choice falls short of explaining how a decision maker's faith progresses over time, particularly for the case of total ambiguity. McCloskey (2006: 154) describes our faith as our identity and notes that it is instilled in the sense that children do not begin with values and beliefs of their own. If all decision makers fall into our no ambiguity case, which is very doubtful, one could argue that we observe differences in religious choice because people have experienced different events, which in turn have produced different sets of beliefs. However, our model does not explain why people would ascribe different expected outcomes to the same religious choice, which is surely the reality, and this is relevant to both our no ambiguity case and the total ambiguity case. A more complete model of religious choice would be dynamic, explaining how these perceived outcomes are developed over time as they are influenced by family, friends, and social contexts.

Another critique is that people may fall between the two extremes of total ambiguity and no ambiguity. We have addressed this in another article (Melkonyan and Pingle, *forth.*). By parameterizing both the degree of pessimism and the degree of ambiguity, one can show that changes in either can, in general, lead a decision maker to switch religions. However, it is also true that more ambiguity places more emphasis on outcomes, as opposed to probabilities. As the total ambiguity case is reached and the influence of probability weights wanes, religious choice becomes highly dependent on which religions offer extreme outcomes, the max-max outcome and max-min outcomes. This explains the sway that concepts such as heaven and hell can play in religious choice.

Quinn (1994) notes that there are nonreligious philosophical reasons for not using the Pascalian decision theoretic approach to choose a religion. He notes that Clifford's credulity argument is that it is wrong always, everywhere, and for anyone to believe anything in the face of insufficient evidence. In this perspective, one should not adopt any particular perspective about what happens after death if one cannot effectively collect the evidence needed to separate truth from fiction. Quinn notes that many people, including Pascal, have made religious choices based on experiences that they have perceived as evidence. Nonetheless, this credulity argument could be used to justify agnosticism. In saying, with regard to religious choice, "You must wager," Pascal appeared to disagree, implicitly indicating that agnosticism is a choice with consequences that must be included in the decision matrix. Rationalizing agnosticism within a decision theoretic framework would therefore seem to be another worthwhile extension of this work.

Finally, this model of religious choice provides testable hypotheses. For example, within Christianity, all else being equal, we would expect "hellfire" evangelism to disproportionately attract pessimists but "heaven-focused" evangelism to disproportionately attract optimists.⁸ By testing the implications of the model, we should obtain understanding about the extent to which people choose their religion in a Pascalian manner. If the evidence indicates that people choose their religion in this manner, the tests might also enable the identification of decision criteria that are used. Alternatively, if the tests indicate that the Pascalian model does not explain choice, they might nonetheless suggest alternative models that explain the different religions people adopt.

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⁸ We thank an anonymous referee for emphasizing the testability of the theory and for offering this example.

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